Correspondence

Noise Performance of Traveling-Wave Masers*

In this communication, a review of the noise performance of traveling-wave masers (TWM) is given. It is shown in particular that when the gain per unit length of structure is low, the equivalent noise temperature of the TWM can become appreciable.

The expression for noise temperature of a TWM has been derived by many investigators, and we shall here use the form given by Siegman,[1]

$$T_m = \frac{G-1}{G}\left[\frac{\alpha_s T_s}{\alpha_s - \alpha_0} + \frac{\alpha_0 T_0}{\alpha_s - \alpha_0}\right], \quad (1)$$

where

- T_s = spin temperature,
- T_0 = structure temperature,
- α_s = gain coefficient per unit length of the TWM,
- α_0 = loss coefficient per unit length of the TWM,
- $G = e^{(\alpha_s - \alpha_0)L}$ = net gain for a TWM of length L.

The usual assumption made in the discussion of TWM is that, generally, $G \gg 1$ and $\alpha_s \gg \alpha_0$. Then it is seen that (1) becomes

$$T_m \cong T_s. \quad (2)$$

Since T_s is generally a fraction of the bath temperature, *i.e.*,

$$T_s \simeq \frac{f_s}{f_p - f_s}, \qquad T_0 = \rho T_0, \quad (3)$$

where f_s = signal frequency and f_p = pump frequency, it is seen that T_m can be very small. The system noise contribution originates in the input transmission line losses and in the follow-up receiver. (These latter quantities are to be added in the usual way.)

These results are a consequence of the assumption of high gain per unit length of structure. This assumption is a reasonable one for ruby masers operating at 1.5 to 2.5°K, but it is not necessarily valid for operation at 4.2°K or higher temperatures. At the latter temperatures it is still possible to obtain high net gain, G; however, the gain per unit length is small and a long structure is necessary. Eq. (1) now takes the approximate form

$$T_m \simeq \frac{\alpha_s T_s}{\alpha_s - \alpha_0} + \frac{\alpha_0 T_0}{\alpha_s - \alpha_0} = T_0 \frac{\rho + \beta}{1 - \beta}, \quad (4)$$

where $\beta = \alpha_0/\alpha_s$.

It is noted from (4) that if the coefficient for net gain per unit length $\alpha_s - \alpha_0$ is small then T_m could become appreciable.

Fig. 1 shows graphically the behavior of T_m/T_0 as a function of β for different values of ρ. The usual approximation is given by taking $\beta = 0$ in Fig. 1.

The foregoing discussion indicates the need for careful design of a TWM when operating at elevated temperatures (4.2°K or higher) which the present state-of-art in closed-cycle refrigerators, unfortunately, requires. Particularly important is the need to reduce the forward loss of the structure.

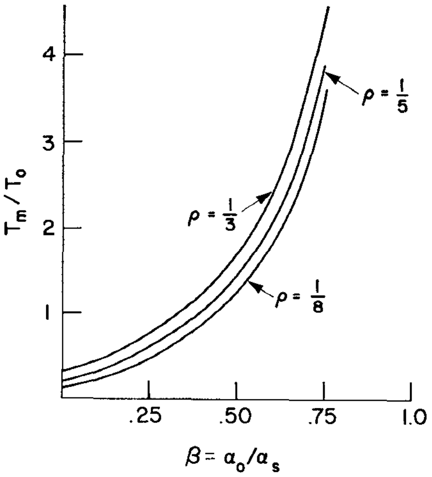

Fig. 1—Ratio of TWM temperature to bath temperature as a function of β (ratio of loss per unit length to electronic gain per unit length).

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Millimeter Frequency Multiplication with an In-Line Harmonic Generator*

When building a crystal frequency multiplier to generate harmonics in the millimeter region, one may improve conversion efficiencies by one of two basic techniques: selecting an improved nonlinear junction, or improving the physical and electrical environment of that junction. This communication will describe an in-line frequency multiplier designed to facilitate changing semiconductors and whiskers to evaluate their efficacy in generating the third and fourth harmonics of a 22-Gc drive signal. In addition, the test results on several different semiconductors will be presented.

In-Line Harmonic Generator

The harmonic generator has been designed for simplicity and rapid whisker and crystal changeability; these goals have been achieved in conjunction with electrical effectiveness by abandoning the crossed-guide structure[1] for a coplanar arrangement, which is evident in the E-plane section of Fig. 1. This multiplier has a WR-42 input waveguide and WR-12 output waveguide. As Fig. 2 shows, these two guides are in line, and perpendicular to them are two tunable shorts located symmetrically with respect to the crystal site. The region between the input and output guides is tapered so that in the immediate vicinity of the crystal, the waveguide has a 0.420×0.050-inch cross section, and the two tuners also have a guide height of 0.050 inch.

The low guide allows the use of a short, relatively rigid whisker which, in comparison with the crossed-guide unit, is very easily replaced, since there is no necessity for threading a long whisker through a small hole between fundamental and harmonic guides. In practice, the short whisker can be changed in a few minutes.

Electrically the in-line frequency multiplier has a slightly different circuit from the more common crossed-guide multiplier. Fig. 3 shows both circuits. The crossed-guide multiplier has a circuit that approximates a series connection of the crystal; the in-line circuit is approximately a shunt connection of the crystal and the input and output circuits. The radical aspect ratio of the waveguide (8.4:1) in the immediate vicinity of the multiplying junction leads to very low integrated input and output impedances,[2] of the order of 90 ohms. According to the formulas of Leeson and Weinreb[3] applied to a typical point contact crystal, these guide impedances are of the right order of magnitude for matching reactive generation of the third and fourth harmonics when the second is suppressed. However, this match depends upon the power level.

The low guide height also limits the effects of standing waves on the whisker. Measurements made by Swago on a large in-line harmonic generator of variable guide height showed high and consistent output for heights less than approximately $\frac{3}{8}$ of the harmonic wavelength; beyond that, the output was an erratic function of the guide height.[4] In the frequency multiplier described here, the height is 0.28 λ for the third

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[2] E. C. Jordan, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., Englewood Cliffs, N. J., p. 281; 1950.

[3] D. B. Leeson and S. Weinreb, "Frequency multiplication with non-linear capacitors—a circuit analysis," Proc. IRE, vol. 47, pp. 2076–2084; December, 1959.

[4] A. W. Swago, "Crystal Multipliers," Quarterly Progress Report Number 11, Research and Investigation Leading to Methods of Generating Radiation in the 100 to 1000 Micron Range of the Spectrum, University of Illinois, Urbana, USAEC Contract AT(11-1)-392, p. 46; March 31, 1959.

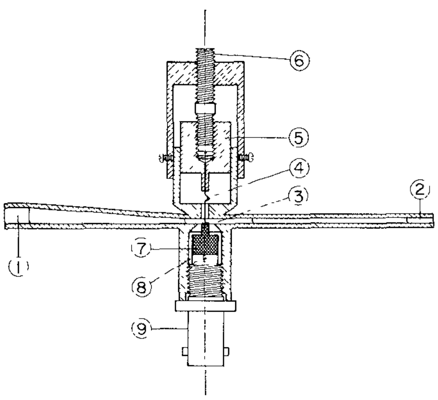

Fig. 1—An E-plane section of the in-line harmonic generator, showing: 1) input waveguide, 0.420 ×0.170 in, 2) output waveguide, 0.122 ×0.061 in, 3) multiplying region, 0.420 ×0.050 in, 4) whisker, 5) whisker holder, 6) differential screw, 7) wafer holder, 8) Teflon insulator, 9) BNC connector.

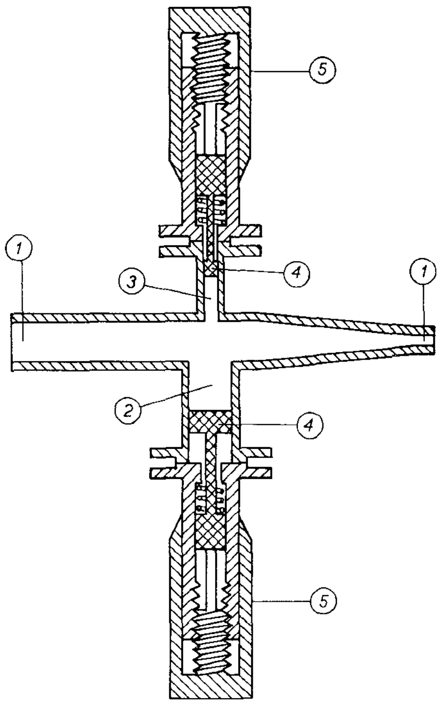

Fig. 2—An H-plane section of the in-line harmonic generator, showing: 1) input and output waveguides, 2) fundamental tuning guide, 0.420 ×0.050 in, 3) harmonic tuning guide, 0.122 ×0.050 in, 4) tunable shorts, 5) micrometer heads for adjusting shorts.

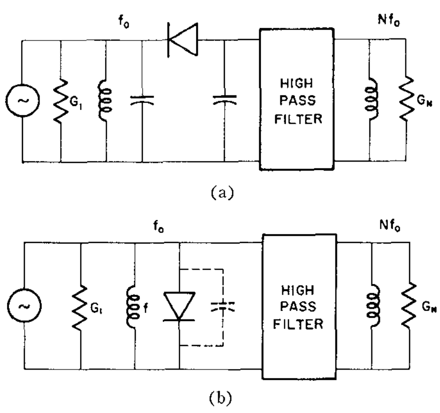

Fig. 3—Approximate equivalent circuits for the two types of frequency multiplier: (a) the series circuit associated with the crossed-guide multiplier, (b) the shunt circuit associated with the in-line frequency multiplier.

harmonic, and 0.37 λ for the fourth. Finally, eliminating the whisker threading problem found in the crossed-guide multiplier also eliminates a short, potentially lossy coaxial section in the crystal current path.

Multiplier and Materials Evaluation

The in-line frequency multiplier has been operated using a VA 241 as the source of 22-Gc power. Both the input and output ports of the multiplier were tuned with E-H tuners in order to maximize the efficiency of the system. Absolute conversion efficiencies were measured by using bolometers, though crystals were used to determine relative power output levels. During adjustment, whisker-crystal contact was monitored either by a crystal curve tracer or by the rectification of an incident microwave signal. Using the techniques implied by this discussion, conversion efficiencies were determined at various power levels for several different semiconducting crystals; these results are summarized in Table I and Fig. 4.

TABLE I

Harmonic Generating Character of Various Junctions in the In-Line Frequency Multiplier

Semiconductor Wafer Material	Conversion Efficiency for Third Harmonic db	Maximum Drive Power mw
Zn-compensated Te-doped GaAs. (111) Oriented. In-diffused 40 min. at 690°C.	−37*	< 90
Uncompensated Te-doped GaAs. (111) oriented.	−43*	
B-138. Sn-doped GaAs (111) oriented. Epitaxial layer approximately 1.5 micron.‡	−37*	< 80
Si from 1N26	−36*	>140
Si from 1N53	−32†	>140

* Measured with 70-mw input power.
† This figure was actually measured to be 33 db at 135 mw, but is converted by the use of the 1N53 curve in Fig. 4 to read 32 db with 70-mw input power.
‡ This crystal was supplied by J. C. Irvin of Bell Telephone Labs.

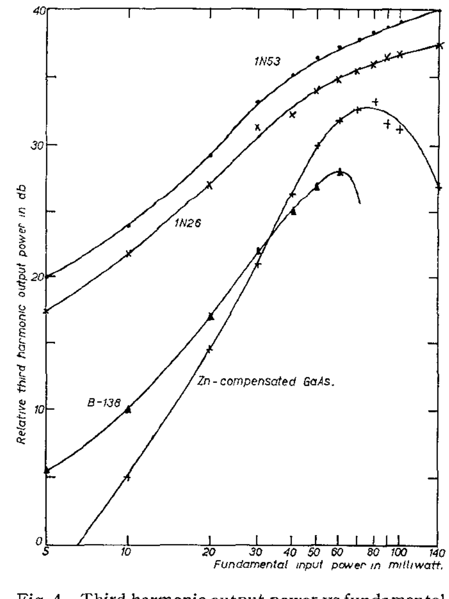

Fig. 4—Third harmonic output power vs fundamental input power for several different semiconductor wafers used in the in-line harmonic generator.

Once it was ascertained that the multiplier itself was working satisfactorily by testing the harmonic generation with silicon junctions, tests were made to see if other semiconductors might prove more effective for millimeter wave frequency multiplication. Because of its large band gap and large ratio of mobility to dielectric constant, Gallium Arsenide (GaAs) appears promising as a high frequency harmonic generator.[5] It is desirable to have GaAs highly doped except for a thin depleted layer at the surface.[6,7] Gallium Arsenide was tested with uniform doping and with the depletion layer approximated by diffusing Zn into the Te doped crystal, forming a compensation layer. The crystal had a resistivity of 10^{-3} ohm cm, and the diffusion took 40 minutes at 690°C. It did effect an improvement in the harmonic generation efficiency, but still, the GaAs tested thus far has proven inferior to silicon removed from 1N26 and 1N53 crystal cartridges.

It was generally found that the diodes with the best conversion efficiencies were biased several volts beyond cutoff. Other diodes performed best when biased a few hundred millivolts in the forward direction, but the harmonic generation efficiencies of the forward-biased units were never observed to exceed the best efficiencies of the back-baised diodes. Certain diodes, of both Si and GaAs, exhibited peaks for both forward and reverse bias. These conditions represent two different regimes of operation, more or less ideal nonlinear capacitative operation with reverse bias, and a combination of nonlinear conduction and nonlinear capacitance with forward bias. Within the scope of this study, the nominally lossless mode of operation has shown itself to be superior.

The multiplier itself has given results which are comparable with those achieved with the crossed-guide frequency multiplier, although the efficiencies are not superior to the best reported for crossed-guide units. Particularly when viewed in terms of evaluating different nonlinear junctions, the in-line multiplier, with competitive electrical characteristics, gains superiority through the ease with which the crystal and whisker may be replaced.

An improvement in the operation of this frequency multiplier might be effected by either of two changes. Because harmonic TE_{10} and TE_{30} waves may be propagated toward the fundamental power source, a reactive low pass filter would be capable of increasing the output at least 3 db. From the Swago guide height work,[4] it may be inferred that further lowering of the waveguide in the vicinity of the crystal would have a beneficial effect, especially for the fourth harmonic.

Acknowledgment

The authors wish to acknowledge the support and encouragement of Professor F. S. Barnes, who supervised the research. Helpful discussions with D. Maley, R. Day

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TM and TE Mode Surface Waves on Grounded, Anisotropic, Inhomogeneous, Lossless, Dielectric Slabs*

J. H. Richmond has given the WKB solutions for the field distribution of surface waves on inhomogeneous, isotropic, plane layers.[1] It is the purpose of this letter to extend his work to include a simple anistropy in the dielectric constant by considering a diagonalized relative permittivity tensor with components $\epsilon_x(x)$, $\epsilon_y(x)$, and $\epsilon_z(x)$. The geometry is the same as before[1] except that a perfectly conducting plane is now positioned at $x=0$. For easy reference we have used the same notation as Richmond, except where specified otherwise. Compactness in notation has been achieved by expressing the integrations from 0 to x and by considering the x variations outside the slab to be $\exp\{-\alpha(x-a)\}$.

The TM solutions for the x variations of the field components are given by

$$H_y = \begin{cases} \exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \dfrac{g(x)}{g(a)}\cdot\dfrac{\cos R(x)}{\cos R(a)}, & \text{in region II,} \end{cases} \tag{1}$$

$$E_z = \begin{cases} \dfrac{j\alpha}{\omega\epsilon_0}\cdot\exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \dfrac{j\alpha}{\omega\epsilon_0}\cdot\dfrac{g(a)}{g(x)}\cdot\dfrac{\sin R(x)}{\sin R(a)}, & \text{in region II,} \end{cases} \tag{2}$$

where

$$g(x) = \left[\frac{\epsilon_z(x)}{r(x)}\right]^{1/2}, \tag{3}$$

$$r(x) = \left[\frac{\epsilon_z(x)}{\epsilon_x(x)}\cdot(k^2\epsilon_x(x) - h^2)\right]^{1/2}, \tag{4}$$

and

$$R(x) = \int_0^x r(x)dx. \tag{5}$$

It is observed that H_y is normalized to unity at the air-slab interface. Also, we have considered the relative permeability of the slab to be unity.

* Received August 26, 1963.

[1] J. H. Richmond, "Propagation of surface waves on an inhomogeneous plane layer," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 554–558; November, 1962.

The transcendental equation for the propagation constant h is given by

$$r(a)\tan R(a) = \alpha\epsilon_z(a). \tag{6}$$

Eq. (6), for a constant, scalar permittivity, reduces to (39) in a standard reference.[2]

The x variations for the TE modes are summarized as

$$E_y = \begin{cases} \exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \left[\dfrac{q(a)}{q(x)}\right]^{1/2}\cdot\dfrac{\sin Q(x)}{\sin Q(a)}, & \text{in region II,} \end{cases} \tag{7}$$

$$H_z = \begin{cases} \dfrac{\alpha}{j\omega\mu_0}\exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \dfrac{\alpha}{j\omega\mu_0}\cdot\left[\dfrac{q(x)}{q(a)}\right]^{1/2}\cdot\dfrac{\cos Q(x)}{\cos Q(a)}, & \text{in region II,} \end{cases} \tag{8}$$

where

$$q(x) = [k^2\epsilon_y(x) - h^2]^{1/2} \tag{9}$$

and

$$Q(x) = \int_0^x q(x)dx. \tag{10}$$

The determining equation for the propagation constant h is

$$q(a)\cot Q(a) = -\alpha, \tag{11}$$

which, for a constant, scalar permittivity, agrees with (46b) in a well-known text.[2]

It is to be noted that the solutions are valid only for slowly varying permittivities, or when

$$\left|\frac{r'(x)}{r^2(x)} - \frac{\epsilon_z'(x)}{r(x)\epsilon_z(x)}\right| \ll 2, \quad \text{TM,} \tag{12}$$

$$\left|\frac{q'(x)}{q^2(x)}\right| \ll 2, \quad \text{TE,} \tag{13}$$

where the prime denotes differentiation with respect to x. We find different restrictions on the TM and TE cases because the wave equations are different for the two cases, being given by

$$H_y'' - \frac{\epsilon_z'(x)}{\epsilon_z(x)}H_y' + \frac{\epsilon_z(x)}{\epsilon_x(x)}(k^2\epsilon_x(x) - h^2)H_y = 0, \quad \text{TM,} \tag{14}$$

and

$$E_y'' + (k^2\epsilon_y(x) - h^2)E_y = 0, \quad \text{TE.} \tag{15}$$

Expressions (12) and (13) can be verified by considering, in detail, standard WKB solutions of the Schroedinger equation.[3]

Conventionally,[1,4] we have neglected derivatives of $g(x)$ and $q(x)$ in finding E_z for the TM case and H_z for the TE case, respectively.

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[2] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Company, Inc., New York, N. Y., ch. 11; 1960.

[3] L. I. Schiff, "Quantum Mechanics," McGraw-Hill Book Company, Inc., New York, N. Y., pp. 184–193; 1955.

[4] L. M. Brekhovskikh, "Waves in Layered Media," Academic Press, New York, N. Y., p. 196; 1960.

Application of Corner Mirrors for Ultramicrowave Interferometers*

The Michelson type and Fabry-Perot type interferometers are often used in the field of ultramicrowaves; the latter replace conventional cavity resonators in the millimeter and submillimeter wave range.[1]

The main problem connected with these devices is the design of suitable mirrors with low loss and adequate reflectivity. So far, planar or spherical mirrors are used.[2] Adjustment of these mirrors is critical and tedious; hence, the use of optical collimation methods is recommended.[3] These difficulties could be remarkably reduced by application of metallic mirrors in the form of rectangular prism corners, *i.e.*, so-called corner mirrors (see Fig. 1).

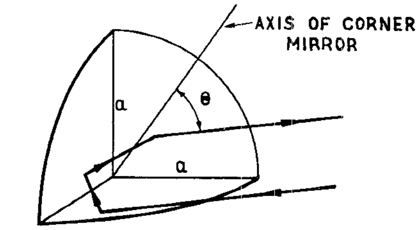

Fig. 1—Cubic corner mirror.

They take advantage of a very old principle of optics and have also been used sometimes in connection with radar techniques.[4] The adjustment of a corner mirror, which consists of three mutually perpendicular metallic plane mirrors, is in principle uncritical. For a correct design of a mirror, it is sufficient to secure the stable position of its peak. Then a beam of electromagnetic waves falling on a mirror at an angle of $\Theta < 35.26°$, after a triple reflection, will return antiparallelly to the falling beam. (For real mirrors having a finite ratio a/λ, a respectively smaller value of the angle Θ may be utilized.[1])

The interferometers utilizing corner mirrors could be designed in the form given in Figs. 2 and 3. The set in Fig. 2 differs from the conventional Michelson interferometer only by the type of the mirrors used, and, therefore, it does not require further discussion.

In the Fabry-Perot interferometer, one must secure suitable coupling with the cavity. For that purpose, one wall of the metallic corner should be half-transparent; a metallic perforated wall would be a good solution.[3] The Fabry-Perot interferometer with corner mirrors can be made in a number of variants (see Fig. 3). The set in Fig. 3(a) differs from that used in the past only by the type of mirrors used and additional

* Received August 16, 1963.

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